I. INTRODUCTION

The problem of determining closest packing dates back at least as far as the argument in the marketplace of old of whether the customer was getting "good measure" in his purchase. St. Luke the evangelist writes in Chapter VI, Verse 38: "Give, and it shall be given unto you; good measure, pressed down, and shaken together, and running over, shall men give into your bosom. For with the same measure that ye mete withal it shall be measured to you again."

The purpose of this thesis is to determine the closest possible packing of equal spheres, and to investigate some related problems in this area. To accomplish this purpose, first the two-dimensional problem of determining the closest packing of circles in a plane is considered. Then the analogous problem of packing spheres in Euclidean three-space is investigated, using as a transition between the two problems the sphere cloud concept of L. Fejes T6th.

Among the related problems is that of whether or not it is possible for thirteen spheres of equal size to be in contact with a fourteenth sphere. Two proofs are presented to show that such contact is indeed impossible, the first proof using the points of contact to form a network, and the second using the concept of central projection.

Finally, a problem is presented which generalizes to the idea of determining the number of (N + I)-vertex figures in

2. CLOSEST PACKING OF CIRCLES

We will consider one packing of circles to be closer than another if a (sufficiently large) prescribed region accommodates more circles of the first packing than of the second.

2.1 Circle Packings and Lattices

TO determine the closest packing of circles, Hilbert and Cohn-Vossen (1952) use the idea of lattices. A square lattice is constructed by marking the four corners of a unit square in the plane. We then move the square one unit of length in the direction parallel to one of its sides, and mark the two new points indicated by the corners. We continue in this manner, and imagine the process to be repeated indefinitely, first in

the original direction, and then in the opposite direction. Then we proceed in the directions orthogonal to our original directions, and thus cover the entire plane with points (see Figure I). The totality of these points constitutes the square lattice, and it may be noted that instead of using a unit square to generate the lattice, any parallelogram that can be drawn on the lattice such that no lattice points are within its boundaries, and no lattice points lie on its boundaries except for vertices, may be used to generate the square lattice.

Now we will consider a special case of the general "unit lattices," that Is, lattices that can be constructed from an arbitrary parallelogram of unit area in the manner in which we constructed the square lattice from the unit square. For